

Raman scattering on phonon–plasmon coupled modes in magnetic fields

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Raman scattering on phonon–plasmon coupled modes in high magnetic fields is considered theoretically. The calculations of the dielectric function were performed in the long-wave approximation for the semiclassical and ultra-quantum magnetic fields taking into account the electron damping and intrinsic lifetime of optical phonons. The Raman scattering has resonances at the frequencies of coupled modes as well as at multiples of the cyclotron frequency. The dependence of the Raman cross section on the carrier concentration is analyzed.

I. INTRODUCTION

Phonon-plasmon coupled modes arise as a result of the interaction of optical phonons with charge carriers in semiconductors. These modes are usually observed in the absence of a magnetic field, for instance, in Raman scattering [1]. Interest in these modes is related not only to applications in optics but also to principal questions of condensed-state physics, in particular, to the possibility of determining the magnitude of electron-phonon interaction with their help. The first experimental results for the Raman scattering on phonon-plasmon coupled modes in magnetic fields were recently obtained [2], but only the preliminary theoretical paper [3] has been published up to now.

The longitudinal optical phonons interact strongly with free carriers because of an electric field accompanied in the lattice vibrations. Let us estimate the parameters of the corresponding electron-phonon system. The frequency of an emitted or absorbed optical phonon ω is usually equal to several hundreds of degrees, for example, this is 35 mV in GaAs. If the light scattering is excited by the laser frequency $\omega_i = 1.5$ eV, the momentum transfer k from the incident radiation has the order of $\omega_i/c \sim 10^5$ cm⁻¹. Then, the conditions $\omega/c \ll k \ll \omega/v_F$ are fulfilled, where the Fermi velocity is $v_F \sim 0.5 \times 10^8$ cm/c in the case when the statistics of carriers is degenerate. The left-hand side of the above inequality means that the electric field associated with the phonon vibration of frequency ω is really static and therefore the electric field is longitudinal with respect to the wavevector \mathbf{k} . The right-hand side of the inequality allows the electron contribution to the dielectric function $\varepsilon(\mathbf{k}, \omega)$ to be calculated using a series expansion in terms of the dispersion parameter kv/ω . We shall see that at least the lowest-order correction should be held, because it has a resonant character. As for the lattice contribution to $\varepsilon(\mathbf{k}, \omega)$, it can be taken at $k = 0$, since the phonon dispersion is negligible for such the momentum.

The effect of magnetic field on optical phonons is most essential if the cyclotron frequency of carriers ω_c has the order of the optical phonon frequency. This means that the magnetic field must reach 20 T for effective carrier mass of 0.063 m_0 in the same GaAs. Changing the carrier concentration, we can observe both the semiclassical regime, when the cyclotron frequency is much less

than the Fermi energy ($\omega_c \ll \varepsilon_F$), and the ultra-quantum regime, when $\omega_c > \varepsilon_F$.

Two circumstances should be noted. Firstly, both the frequency of the coupled modes and the width of the corresponding resonance are of interest in the Raman light scattering. Therefore, the contribution of carriers to the dielectric susceptibility must be calculated with regard to their damping. The spatial dispersion of susceptibility in a magnetic field under these conditions has not been calculated so far. Secondly, in addition to the Coulomb interaction of carriers with the longitudinal phonon vibrations, which is taken into account by the dielectric function, there exists a deformation interaction with both the LO and TO modes, which bears the Fröhlich name in theoretical works. It arises because of the nonadiabaticity of electron-phonon systems and leads to a certain (as small as the nonadiabaticity parameter) renormalization of phonon frequencies. In the absence of a magnetic field, this renormalization has been recently considered in [4] - [6], and it will not be taken into account here.

The structure of the paper is the following. A short outline of the theory of Raman scattering on the phonon-plasmon coupled mode is given in Sec. II. Then the dielectric function of the system in semiclassical and ultra-quantum regimes is evaluated in Secs. III and IV. The theoretical Raman spectra are presented for the various magnetic fields and carrier concentrations in Sec. V. Finally, the conclusions are summarized in Sec. VI.

II. INELASTIC LIGHT SCATTERING ON PHONON–PLASMON COUPLED MODES

Let us consider the Raman scattering on optical vibrations in a polar lattice with free carriers. We use the notations b_j for the phonon displacements of the branch j . The subscript j denotes the various phonon modes: longitudinal or transverse ones. More precisely, the subscript j indicates the different phonon representations which can be degenerate. The transformation properties of the coupling constants g_j are determined by this representation. The LO vibrations are accompanied by the internal electric field $\mathbf{E}(\mathbf{r}, t)$. We consider the phonon displacements and electric field as classical variables because the magnetic field affects quantum-mechanically only the free carriers.

The effective Hamiltonian describing the inelastic light scattering on phonon-plasmon modes can be written as

$$\mathcal{H} = \frac{e^2}{mc^2} \int d^3r \mathcal{N}(\mathbf{r}, t) U(\mathbf{r}, t), \quad (1)$$

where

$$\mathcal{N}(\mathbf{r}, t) = g_j b_j(\mathbf{r}, t) + g_E E(\mathbf{r}, t) \quad (2)$$

is a linear form of variables b_j and E . We do not include the pure electronic Raman scattering by free carriers, which has been considered in many papers (see, e.g. [7], [8]).

The notation $U(\mathbf{r}, t)$ is introduced for a product of the vector-potentials of incident and scattered photons:

$$A^{(i)}(\mathbf{r}, t) A^{(s)}(\mathbf{r}, t) = U(\mathbf{r}, t) = \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)] U(\mathbf{k}, \omega),$$

where the momentum and frequency transfers $\mathbf{k} = \mathbf{k}^{(i)} - \mathbf{k}^{(s)}$, $\omega = \omega^{(i)} - \omega^{(s)}$. The polarization vectors $e_\alpha^{(i)}$ and $e_\beta^{(s)}$ of the incident and scattered photons are included in the coupling constants as well as the polarization vector of $\mathbf{E}(\mathbf{r}, t)$ is included in the coupling constant g_E . The estimation of the coupling constants gives $g_j \sim /a^4$, $g_E \sim 1/ea$ where a is the lattice parameter.

If we evaluate the generalized susceptibility $\chi(\mathbf{k}, \omega)$ in the linear response to the force $U(\mathbf{k}, \omega)$

$$\mathcal{N}(\mathbf{k}, \omega) = -\chi(\mathbf{k}, \omega) U(\mathbf{k}, \omega), \quad (3)$$

we obtain the Raman cross-section

$$\frac{d\sigma}{d\omega^{(s)} d\Omega^{(s)}} = \frac{2k_z^{(s)} \omega^{(s)}}{\pi c (1 - \exp(-\omega/T))} \left(\frac{2e^2}{c\hbar m \omega^{(i)}} \right)^2 \times |U(\mathbf{k}, \omega)|^2 \text{Im} \chi(\mathbf{k}, \omega), \quad (4)$$

where $k_z^{(s)}$ is the normal to the sample surface component of the scattered wave vector in vacuum.

One note should be made here. Of course, any sample has the surface. The surface effects in the Raman scattering was considered in the work [9] and they are omitted in the derivation of Eq. (4). Furthermore, the incident and scattered fields do not penetrate into the bulk due to the skin effect. For the optical range of the incident light, we have the normal skin-effect conditions. Then we integrate in Eq. (4) the distribution $|U(\mathbf{k}, \omega)|^2$ over the normal component k_z . As shown in the paper [9], the integration of $|U(\mathbf{k}, \omega)|^2$ gives a factor $1/\zeta_2$, where ζ_2 is expressed in terms of the wave-vector components inside semiconductor: $\zeta_2 = \text{Im}(k_z^{(i)} + k_z^{(s)})$. The Raman cross section (4) obtained is dimensionless. It represents a ratio of the inelastic scattered light energy to the incident energy.

The equation of motion for the LO mode b_{LO} has the form

$$(\omega_{\text{TO}}^2 - \omega^2) b_{\text{LO}}(\mathbf{k}, \omega) = \frac{Z}{M'} E(\mathbf{k}, \omega) - \frac{g_{\text{TO}} U(\mathbf{k}, \omega)}{M' N}, \quad (5)$$

where N is the number of unit cells in 1 cm^3 , M' is the reduced mass of the unit cell, and Z is the effective ionic charge. Notice that the optical phonons always have the so-called natural width $\Gamma \sim \omega_0 \sqrt{m/M}$. The natural width results from decay processes into two or more acoustic and optical phonons. In the final expressions, we will substitute $\omega_{\text{TO}}^2 - \omega^2 \rightarrow \omega_{\text{TO}}^2 - i\omega\Gamma - \omega^2$. The equation (5) is applied as well to the transverse phonons, but the electric field has to be neglected in the case of the TO phonons.

The electric field $\mathbf{E}(\mathbf{r}, t)$ can be obtained from the Poisson equation $\text{div} \mathbf{D} = 0$. There are several contributions in the induction D : (1) the polarization $\alpha E(\mathbf{r}, t)$ of the filled electron bands, (2) the lattice polarization $NZ b_{\text{LO}}(\mathbf{r}, t)$, (3) the contribution of free carrier density $\rho = -\text{div} \mathbf{P}_e$, and (4) the term $P = -\partial \mathcal{H} / \partial E = -g_E U$ explicitly results from the Hamiltonian, Eqs. (1), (2). Collecting all these terms into the Poisson equation, we find

$$\varepsilon_\infty E(\mathbf{k}, \omega) + 4\pi N Z b_{\text{LO}}(\mathbf{k}, \omega) + \frac{4\pi i e}{k} \rho(\mathbf{k}, \omega) - 4\pi g_E U(\mathbf{k}, \omega) = 0, \quad (6)$$

where the high-frequency dielectric constant $\varepsilon_\infty = 1 + 4\pi\alpha$.

We shall calculate the carrier density $\rho(\mathbf{k}, \omega)$ in the following sections and find the electron contribution in the dielectric function $\varepsilon_e(\mathbf{k}, \omega) = \varepsilon_\infty + 4\pi i e \rho(\mathbf{k}, \omega) / k E$. Then Eq. (6) takes the form

$$\varepsilon_e E(\mathbf{k}, \omega) + 4\pi N Z b_{\text{LO}}(\mathbf{k}, \omega) - 4\pi g_E U(\mathbf{k}, \omega) = 0. \quad (7)$$

Solving Eqs. (5), (7) and using Eq. (2), we find $\mathcal{N}(\mathbf{k}, \omega)$ and obtain the susceptibility defined by Eq. (3):

$$\chi(\mathbf{k}, \omega) = 4\pi g_E^2 \frac{\varepsilon_e(\mathbf{k}, \omega) C^2 \omega_{\text{TO}}^4 / \varepsilon_\infty \omega_{pi}^2 - \Delta - 2C\omega_{\text{TO}}}{\varepsilon_e(\mathbf{k}, \omega) \Delta + \varepsilon_\infty \omega_{pi}^2}, \quad (8)$$

where we set $\Delta = \omega_{\text{TO}}^2 - \omega^2 - i\omega\Gamma$, the plasma frequency of ions $\omega_{pi}^2 = 4\pi N Z^2 / \varepsilon_\infty M'$, and the Faust-Henry coefficient $C = g_{\text{TO}} Z / g_E M' \omega_{\text{TO}}^2$.

We see from Eq. (8), that the poles of the general susceptibility coincide with the zeros of the total dielectric function

$$\varepsilon(\mathbf{k}, \omega) = \varepsilon_e(\mathbf{k}, \omega) + \frac{\varepsilon_\infty \omega_{pi}^2}{\omega_{\text{TO}}^2 - \omega^2 - i\omega\Gamma}. \quad (9)$$

Therefore, the peaks of the Raman cross section give the frequencies of coupled modes determined by the condition $\varepsilon(\mathbf{k}, \omega) = 0$.

In order to calculate the generalized susceptibility, Eq. (8), we must find the dielectric function $\varepsilon_e(\mathbf{k}, \omega)$, i.e. the free carrier density $\rho(\mathbf{k}, \omega)$.

III. THE DIELECTRIC FUNCTION IN SEMICLASSICAL REGIME

Consider the geometry when the magnetic-field effect is most important: the magnetic field \mathbf{H} is directed along the axis z , whereas the wavevector \mathbf{k} and the corresponding electric field \mathbf{E} are directed in the perpendicular direction x . In semiclassical conditions $\omega_c \ll \varepsilon_F$, we can use the Boltzmann equation

$$-i(\omega - kv_x)f + \omega_c \frac{df}{d\varphi} = v_x - (f - \langle f \rangle)/\tau, \quad (10)$$

written in the τ -approximation within the momentum variables: the component p_z along the magnetic field, the angle φ of rotation around the magnetic field, and the energy ε . The element of phase volume in these variables is $md\varepsilon dp_z d\varphi$. For a quadratic electron spectrum, we can assume, for example, that $v_x = v_\perp(p_z) \sin \varphi$. The factor $e\mathbf{E}\delta(\varepsilon - \varepsilon_F)$, containing the charge, the electric field, and the Dirac function, is separated out of the distribution function f . Angle brackets designate the average over the Fermi surface

$$\langle \dots \rangle = \int (\dots) m dp_z d\varphi / \int m dp_z d\varphi.$$

The term with averaging in the Boltzmann equation is necessary for the fulfillment of the conservation law of the electric charge.

We assume $kv/\omega \ll 1$ and expand the solution to Eq. (10) in powers of this parameter to the second order $f = f_0 + f_1 + f_2$. Because $\langle v_x \rangle$ vanishes in integrating over φ , it is seen from Eq. (10) that $\langle f_0 \rangle = 0$. Therefore, the system of equations for f_0, f_1 , and f_2 takes the form

$$\begin{aligned} -i\omega^* f_0 + \omega_c \frac{df_0}{d\varphi} &= v_x, \\ -i\omega^* f_j + \omega_c \frac{df_j}{d\varphi} &= \frac{\langle f_j \rangle}{\tau} - ikv_x f_{j-1}, \end{aligned}$$

where $j = 1, 2$ and $\omega^* = \omega + i/\tau$. The zeroth- and first-order approximation, which have to be periodic in the angle φ , are readily found

$$\begin{aligned} f_0 &= \int_{-\infty}^{\varphi} v_x \exp[-i\omega^*(\varphi - \varphi')/\omega_c] = \\ &= \frac{-v_\perp}{2} [(\omega_c - \omega^*)^{-1} e^{i\varphi} + (\omega_c + \omega^*)^{-1} e^{-i\varphi}] \end{aligned} \quad (11)$$

$$f_1 = \int_{-\infty}^{\varphi} (\langle f_1 \rangle / \tau) - ikv_x f_0 \exp[-i\omega^*(\varphi - \varphi')/\omega_c], \quad (12)$$

First, $\langle f_1 \rangle$ and, then, f_1 can be found by averaging both the sides of the last equation. We obtain

$$\begin{aligned} f_1 &= \frac{-ikv_\perp^2}{4(\omega_c^2 - \omega^{*2})} \left[\frac{4i}{3\omega\tau} + 2\frac{v_\perp^2}{v_0^2} \right. \\ &\quad \left. + \frac{(\omega_c + \omega^*)v_\perp^2}{(2\omega_c - \omega^*)v_F^2} e^{2i\varphi} + \frac{(\omega_c - \omega^*)v_\perp^2}{(2\omega_c + \omega^*)v_F^2} e^{-2i\varphi} \right]. \end{aligned} \quad (13)$$

For the second-order approximation, an expression similar to f_1 , Eq. (12) is obtained. The difference is connected with the fact that f_2 is an odd function of velocity, and, therefore, the mean value $\langle f_2 \rangle$ vanishes. It can be found that

$$\begin{aligned} f_2 &= \frac{k^2 v_\perp v_F^2}{8(\omega_c^2 - \omega^{*2})} \left[\left(\frac{4i}{3\omega\tau} + 2\frac{v_\perp^2}{v_0^2} \right) \left(\frac{e^{i\varphi}}{\omega_c - \omega^*} + \frac{e^{-i\varphi}}{\omega_c + \omega^*} \right) \right. \\ &\quad \left. + \frac{(\omega_c + \omega^*)v_\perp^2}{(2\omega_c - \omega^*)v_F^2} \left(\frac{e^{3i\varphi}}{3\omega_c - \omega^*} - \frac{e^{i\varphi}}{\omega_c - \omega^*} \right) \right. \\ &\quad \left. - \frac{(\omega_c - \omega^*)v_\perp^2}{(2\omega_c + \omega^*)v_F^2} \left(\frac{e^{-i\varphi}}{\omega_c + \omega^*} - \frac{e^{-3i\varphi}}{3\omega_c + \omega^*} \right) \right]. \end{aligned} \quad (14)$$

It is simpler to calculate the current $j(\mathbf{k}, \omega)$ instead of the electric charge, using the conservation law $\rho(\mathbf{k}, \omega) = kj(\mathbf{k}, \omega)/\omega$. The contribution of carrier into the conductivity is

$$\sigma = e^2 \int v_x f m \frac{2d\varphi dp_z}{(2\pi\hbar)^3}. \quad (15)$$

Because the first-order approximation f_1 , Eq. (13) is even with respect to velocity, it makes no contribution to the conductivity. Using the zeroth-order, Eq. (11) and the second-order, Eq. (14) approximations, we find the carrier conductivity, Eq. (15) and the susceptibility $4\pi i\sigma/\omega$. With regard to the contribution of filled bands, the electron susceptibility can be written as

$$\begin{aligned} \varepsilon_e(\omega, k) &= \varepsilon_\infty \left\{ 1 - \frac{\omega_{pe}^2 \omega^*}{\omega(\omega^{*2} - \omega_c^2)} \right. \\ &\quad \left. - \frac{3\omega_{pe}^2 \omega^* k^2 v_F^2}{5\omega(\omega^{*2} - \omega_c^2)^2} \left(1 + \frac{5i}{9\omega\tau} + \frac{3\omega_c^2}{\omega^{*2} - 4\omega_c^2} \right) \right\}. \end{aligned} \quad (16)$$

The expression obtained remains valid both in the absence of a magnetic field ($\omega_c = 0$) and in the collisionless limit ($\tau = \infty$). If both these conditions are fulfilled, Eq. (16) converts to the known expression with the true coefficient $3/5$. This coefficient, as well as the others in Eq. (16), was calculated here for the quadratic electron spectrum; however, the dependence itself on the frequency ω , magnetic field, and damping is retained in the general case. It is easy to see that the highest resonances $n\omega_c$ contribute to the dielectric function the terms of the order $(kv_F/\omega_c)^n$ with the even n .

The dielectric function of a nondegenerate electron plasma was obtained in Ref. [10]

$$\varepsilon(\omega, k) = 1 - \frac{\omega_{pe}^2 \omega^*}{\omega} \left[\frac{1 - \lambda}{\omega^{*2} - \omega_c^2} + \frac{\lambda}{\omega^{*2} - 4\omega_c^2} \right], \quad (17)$$

where $\lambda = (kv_{th}/\omega_c)^2$, and v_{th} is the thermal velocity of electrons. At $\tau = \infty$ and $\lambda = (kv_F/\omega_c)^2/5$, it coincides with the electronic term in Eq. (16).

IV. ULTRA-QUANTUM MAGNETIC FIELDS

Now we consider the very high magnetic field when only one lowest electron level is occupied. In order to

calculate the dielectric function, we use the expansion of the electron density

$$\sum_{s,s'} \psi_s^*(\mathbf{r}) \psi_{s'}(\mathbf{r}) a_s^+ a_{s'} \quad (18)$$

in terms of the eigenfunctions

$$\psi_{s'}(\mathbf{r}) = \chi_n(x - cp_y/eH) e^{i(p_z z + p_y y)/\hbar},$$

where s is the total set of the quantum numbers $s = \{n, p_y, p_z\}$ and χ_n can be expressed in terms of the Hermite polynomials. The magnetic field is chosen in the z -direction.

Using the equation of motion for the operator $a_s^+ a_{s'}$ in the weak field $\phi(\mathbf{r})$, we find the linear response

$$\langle 0 | a_s^+ a_{s'} | 0 \rangle = \frac{e \phi_{s',s} (f_s - f_{s'})}{\hbar \omega - \varepsilon_{s'} + \varepsilon_s}, \quad (19)$$

where $\phi_{s',s}$ is the matrix element of the potential $\phi(\mathbf{r})$, f_s is the Fermi distribution function depending on ε_s and the average is taken over the ground state. Inserting Eq. (19) into Eq. (18), averaging and taking the Fourier transform with respect \mathbf{r} , we obtain the induced charge and then the electronic dielectric function

$$\begin{aligned} \varepsilon_e(\mathbf{k}, \omega) = \varepsilon_\infty - \frac{4\pi e^2}{k^2} \sum_{n,n'} \int \frac{dx dp_y dp_z}{(2\pi\hbar)^2} e^{ik_x x} \\ \times \chi_n(-cp_y/eH) \chi_n(x - cp_y/eH) \\ \times \chi_{n'}(-cp'_y/eH) \chi_{n'}(x - cp'_y/eH) \frac{(f_s - f_{s'})}{\hbar \omega - \varepsilon_{s'} + \varepsilon_s}, \end{aligned} \quad (20)$$

where $p'_y = p_y + k_y$, $\varepsilon_s = (n + 1/2 \pm g/4)\hbar\omega_c + p_z^2/2m^*$, $\varepsilon_{s'} = (n' + 1/2 \pm g/4)\hbar\omega_c + p_z^2/2m^*$. In the following, we assume that the g -factor equals 2 as for the free electron.

Let us introduce the new variables: $x - cp_y/eH \rightarrow x$, $-cp_y/eH \rightarrow x'$. Then the integral (20) takes the form:

$$\begin{aligned} \varepsilon_e(\mathbf{k}, \omega) = \varepsilon_\infty - \frac{4\pi e^3 H}{ck^2} \sum_{n,n'} \int \frac{dx dx' dp_z}{(2\pi\hbar)^2} e^{ik_x(x-x')} \\ \times \chi_n(x) \chi_n(x') \chi_{n'}(x - ck_y/eH) \chi_{n'}(x' - ck_y/eH) \\ \times \frac{(f_s - f_{s'})}{\hbar \omega - \varepsilon_{s'} + \varepsilon_s}. \end{aligned} \quad (21)$$

Because \mathbf{H} is perpendicular to \mathbf{k} , we can choose $k_y = k_z = 0$. Then we get under the integral (21) the matrix element $I_{n,n'} = (e^{ik_x x})_{n,n'}$ squared.

The wave function $\chi_n(x)$ changes over the magnetic length $a_H = \sqrt{\hbar/m^*\omega_c}$. We have $k \ll 1/a_H$ for the high magnetic fields. Therefore we can evaluate the integral $I_{n,n'}$ expanding the exponent in powers of k_x and using the known matrix elements $x_{n,n-1} = x_{n-1,n} = \sqrt{n\hbar/2m^*\omega_c}$. Due to the Fermi factors $f_s, f_{s'}$, the integrand in Eq. (21) does not vanish only if $n \neq n'$ and one of the numbers n, n' have to be zero in the ultra-quantum limit. Then we are interested in the off-diagonal matrix

elements $I_{n,n'}$. Under these conditions, we obtain to the forth order in k :

$$\begin{aligned} |I_{n,n'}|^2 = \frac{\hbar k^2}{2m^*\omega_c} (n+1) \delta_{n',n+1} \\ - \left(\frac{\hbar k^2}{2m^*\omega_c} \right)^2 \left[\frac{1}{3} (n+1)(2n+3) \delta_{n',n+1} \right. \\ \left. - \frac{1}{4} (n+1)(n+2) \delta_{n',n+2} \right] + (n \leftrightarrow n'). \end{aligned} \quad (22)$$

Substituting Eq. (22) into Eq. (21), taking terms with $n = 0$ or $n' = 0$, and integrating with respect p_z , we find the electron contribution into the dielectric function in the ultra-quantum limit

$$\begin{aligned} \varepsilon_e(\mathbf{k}, \omega) = \varepsilon_\infty - \frac{2e^2 p_F \omega_c}{\pi \hbar^2} \\ \times \left(\frac{1}{\omega^2 - \omega_c^2} - \frac{(ka_H)^2/2}{\omega^2 - \omega_c^2} + \frac{(ka_H)^2/2}{\omega^2 - 4\omega_c^2} \right), \end{aligned} \quad (23)$$

where p_F is the Fermi momentum in the magnetic field. In order to find this value, we assume that the electron concentration

$$\sum_s f_s |\chi_n(x - cp_y/eH)|^2$$

is fixed by the defect concentration n_0 . In the ultra-quantum limit, all the electrons are on the lowest level with $n = 0$. Integrating with respect p_y and p_z , we get

$$n_0 = 2m^*\omega_c p_F / (2\pi\hbar)^2 \quad (24)$$

and Eq. (23) becomes

$$\begin{aligned} \varepsilon_e(\mathbf{k}, \omega) = \varepsilon_\infty - \varepsilon_\infty \omega_{pe}^2 \\ \times \left(\frac{1}{\omega^2 - \omega_c^2} - \frac{(ka_H)^2/2}{\omega^2 - \omega_c^2} + \frac{(ka_H)^2/2}{\omega^2 - 4\omega_c^2} \right), \end{aligned} \quad (25)$$

where $\omega_{pe}^2 = 4\pi n_0 e^2 / m^* \varepsilon_\infty$. Therefore Eq. (25) coincides with Eq. (16), if $1/\tau = 0$ and $v_F^2 = 5\hbar\omega_c/2m^*$. We see, that the parameter $\lambda = (kv_F/\omega_c)^2/5 \sim (k/H)^2$ in the semiclassical case, Eq. (17), whereas the corresponding parameter in the ultra-quantum limit, Eq. (25), is $\lambda' = (ka_H)^2/2 \sim k^2/H$.

The effect of collisions was not taken into account in Eq. (25). It can be done with the help of the simple phenomenology similar to the paper [11]. But for the case of our interest $1/\tau \ll \omega$, when the coupled modes are observed, we can merely replace ω by $\omega + 1/\tau$ in Eq. (25).

V. THEORETICAL RAMAN SPECTRA

The zeros of the dielectric function $\varepsilon(\mathbf{k}, \omega)$, Eq. (9) give the frequencies of the longitudinal modes. With the neglect of the electron τ^{-1} and phonon Γ damping, the

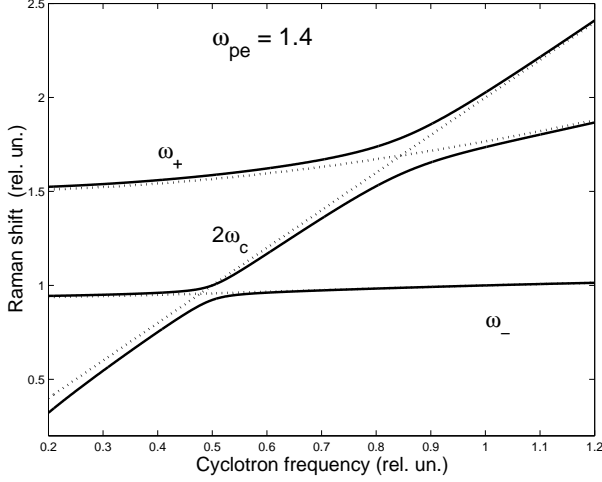


FIG. 1: Frequencies of the phonon-plasmon coupled modes as functions of the carrier cyclotron frequency for the carrier plasma frequency $\omega_{pe} = 1.4$ and $\omega_{LO} = 1.1$; all frequencies are in units ω_{TO} . The solid lines are obtained in solution of the cubic equation (see text), the dotted lines show the results ignoring the k -dispersion.

dielectric function, Eqs. (16) and (25), is real and the frequencies of the corresponding vibrations are also real. In order to determine these frequencies, one has to solve a cubic equation.

To a good approximation one can first omit the small term with k^2 and obtain two eigenfrequencies ω_{\pm}

$$\omega_{\pm}^2 = \frac{1}{2}(\omega_c^2 + \omega_{TO}^2 + \omega_{pi}^2 + \omega_{pe}^2) \pm \frac{1}{2}[(\omega_c^2 - \omega_{TO}^2 - \omega_{pi}^2 + \omega_{pe}^2)^2 + 4\omega_{pi}^2\omega_{pe}^2]^{1/2} \quad (26)$$

by solving the corresponding quadratic equation. The third frequency is found in the vicinity of the pole $\omega = 2\omega_c$ of the omitted term. This approximation is shown in dotted lines in Figs. 1 and 2.

The numerical results including the k -dispersion are shown in solid lines. Now we must distinguish the semiclassical and ultra-quantum regimes. In the last case, the condition $\hbar\omega_c > \varepsilon_F$ is fulfilled and the Fermi energy ε_F depends on the magnetic field according to Eq. (24). Then the condition of ultra-quantum regime is rewritten as $(2eH/\hbar c)^{3/2} > (2\pi)^2 n_0$ for the fixed carrier concentration n_0 . For instance, the ultra-quantum regime is realized for the concentration $n_0 = 10^{17} \text{ cm}^{-3}$ at $H > 7.8 \text{ T}$ and for $n_0 = 10^{18} \text{ cm}^{-3}$ at $H > 36 \text{ T}$. We suppose that the semiclassical condition, Eq. (16) is obeyed in the interval of magnetic fields displaced in Fig. 1, whereas the ultra-quantum regime, Eq. (25) is assumed in Fig. 2.

With the help of Eqs. (4) and (8), we can plot the theoretical Raman spectra for the large, Fig. 3 and low, Fig. 4 electron concentrations. The Faust-Henry coefficient is taken $C = -0.5$ as is usually accepted in the absence of the magnetic field. We see always three peaks: the weak

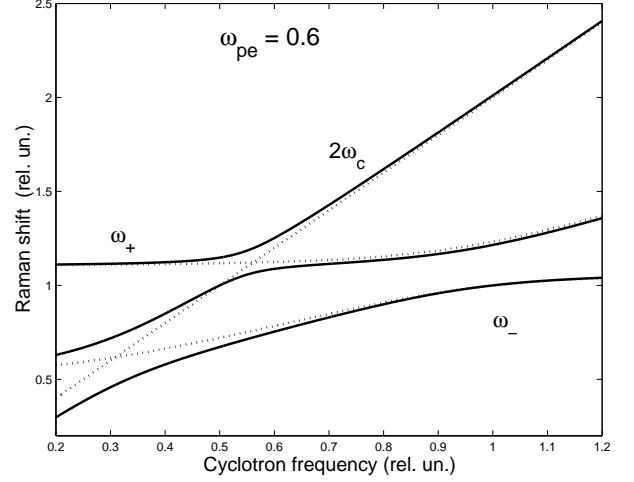


FIG. 2: The same as in Fig. 1 but for $\omega_{pe} = 0.6$.

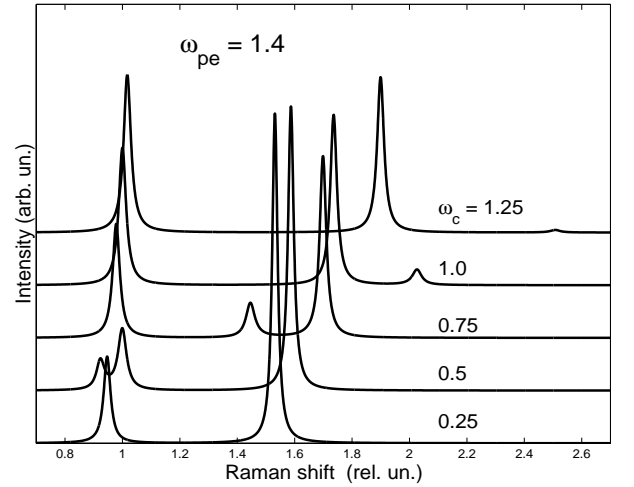


FIG. 3: Calculated Raman spectra for the carrier plasma frequency and cyclotron frequency indicated in figure. All the frequencies are in units ω_{TO} ; the phonon width $\Gamma = 3 \times 10^{-2} \omega_{TO}$, the carrier relaxation rate $\tau^{-1} = 2 \times 10^{-2} \omega_{TO}$, the plasma frequency of ions $\omega_{pi} = 0.4 \omega_{TO}$, and the momentum transfer $k v_F = 0.3 \omega_{TO}$. The cyclotron frequency is supposed to be in the semiclassical regime.

resonance approximately at $\omega = 2\omega_c$ and two resonances corresponding ω_{\pm} . Far from the crossing, the modes have mainly a character of the phonon or plasmon vibrations.

From the splitting in Figs. 1 and 2, we can conclude that the resonance at $\omega = 2\omega_c$ interacts more intensively with the mode of the plasmon character (ω_+ for the large concentration of carriers and ω_- for the low concentration).

The figure 3 (see, also, Fig. 1) clearly demonstrates the result of screening in the case of a relatively large concentration of carriers: the frequency ω_- , which equals the frequency of the longitudinal mode in the absence of

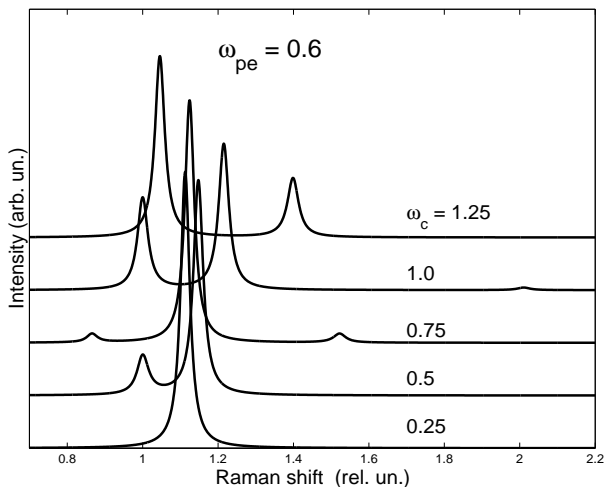


FIG. 4: The same as in Fig. 3 but for the carrier plasma frequency indicated in figure, $\tau^{-1} = 10^{-2}\omega_{TO}$, and for the ultra-quantum regime.

carriers (at the particular choice of parameters, $\omega_{LO} = 1.1\omega_{TO}$), turns out to be close to the frequency of the transverse mode ω_{TO} . The case of the relatively low carrier concentration is illustrated by Figs. 2 and 4. It is seen in Fig. 4 that, in a weak field ($\omega_c = 0.25\omega_{TO}$), the frequency of the longitudinal mode is close (as it must be) to $\omega_{LO} = 1.1\omega_{TO}$. Besides the weak resonance

at $\omega = 2\omega_c$, a phonon-plasmon crossover is seen in the curves for $\omega_c = 0.75\omega_{TO}$ and $1.0\omega_{TO}$. As the magnetic field increases, the weaker plasmon peak, after passing through the phonon one, becomes more intense.

VI. CONCLUSIONS

In this work we investigated the spatial dispersion effect in the Raman scattering on the phonon-plasmon coupled modes. In spite of the small value of corresponding parameters kv_F/ω (for the semiclassical magnetic fields) and ka_H (for the ultra-quantum regime), this effect is observable in the high magnetic fields when the cyclotron resonances intersect the frequencies of the phonon-plasmon coupled modes. The magnetic field provides an additional parameter which gives a possibility to compare the effect of the electron interaction with both the longitudinal and transverse optical modes in doped semiconductors.

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